Equations and Inequalities

Solve the equations and the inequalities:

1. $2x-3=4x-1$
2. $x^2-3x+2=0$
3. $2x+3>5$.
4. $-3x+1\leq x+9$
5. $|x-3|<3$
6. $|2x+3|\geq 1$
7. $|2x+3|=7$
8. $x^2-4x+3<0$
9. $(x-3)(2x+4)\geq 0$
10. $x(x-2)(x-5)>0$

Solutions

1.

$2x-4x=-1+3$

$-2x=2$

$x=2/(-2)$

$x=-1$

2.

Use the quadratic formula or factoring to solve the equation:

$\underline{x^2}-2x-\underline{x}+2=0$

$x(x-1)-2(x-1)=0$

$(x-1)(x-2)=0$

Thus the equation has the solution $x=1$ and $x=2$.

3.

$2x+3>5$

$2x>5-3$

$2x>2$

$x>1$

x€$(1,\infty)$

4.

$-3x+1\leq x+9$

$-3x-x\leq 9-1$

$-4x\leq 8$

$x\geq 8/(-4)$

$x\geq -2$

$x€[-2,\infty)$

5.

$|x-3|<3$

$-3<x-3<3$

$-3+3<x<3+3$

$0<x<6$

$x€(0,6)$

6.

$|2x+3|\geq 1$

$2x+3\geq 1$ or $2x+3\leq -1$

$2x\geq 1-3$ or $2x\leq -1-3$

$2x\geq -2$ or $2x\leq -4$

$x\geq -1$ or $x\leq -2$

$x€(-\infty,-2]$ U $[-1,\infty)$

7.

$|2x+3|=7$

$2x+3 =7$ or $2x+3=-7$

$2x=7-3$ or $2x=-7-3$

$2x=4$ or $2x=-10$

$x=2$ or $x=-5$

$x€\{-5,2\}$

8.

$x^2-4x+3<0$

We first solve $x^2-4x+3=0$
$\underline{x}^2-3x-\underline{x}+3=0$
x(x-1)-3(x-1)=0

(x-1)(x-3)=0

x=1 or x=3

So the solution of the inequality $x^2-4x+3<0$
(x-1)(x-3)<0 is

x€(1,3)

9.
We first solve the equation

(x-3)(2x+4)=0

x-3=0 or 2x+4=0

x=3 or x=-4/2

x=3 or x=-2

So the solution of (x-3)(2x+4)≥0 is

x€(-∞,-2] U [3, ∞)

10.
We first solve the equation x(x-2)(x-5)=0; x=0 or x=2 or x=5

To solve x(x-2)(x-5)>0 we make a table

x	-∞ 0 2 5 +∞
x	------------0++
x-2	--------------------------0++++++++++++++++++++++++++++++
x-5	--0+++++++++++++++++++++++
x(x-2)(x-5)	-------------0+++++++++0-----------0+++++++++++++++++++++++

and we get x(x-2)(x-5)>0 for x€(0,2) U (5,∞)

Lines, circles, ellipses, hyperbolas, parabolas

1. Find an equation of the line that passes through (-1,0) and has slope 6.
2. Find an equation of the line with slope 3 and y-intercept 1.
3. Find an equation of the line that passes through (-1,3) and it is parallel to the line 2x+y=3.
4. Find an equation of the line that passes through (5,-1) and it is perpendicular to the line 4x-8y=16.
5. The equation
$x^2+y^2+5x+6=0$

represents a .... with radius R=.... and center with coordinates x_0=...... and y_0=.......
6. The equation $x=-3y^2+1$ represents a .... symmetric with the ...- axis that opens to the .....
7. The equation

$$y = x^2 - 3x + 2$$

represents a .... that opens ...... and has vertex with coordinates x0=.... and y0=......
8. The equation
$y=3x^{2-}15x+12$
represents a .... with vertex with coordinates x0=..... and y0=...... and x -intercepts x1=..... and x2=......, and y-intercept y1=......

Solutions

1.

From the point-slope formula we have

y-0=6(x+1)

y=6x+6

2.

From the slope-intercept formula we have

y=3x+1

3.

For the line 2x+y=3

y=-2x+3

the slope is -2. Since the two lines are parallel, they have the same slope -2.

From the point-slope equation we have

y-3=-2(x+1)

y=-2x-2+3

y=-2x+1

4.

For the line 4x-8y=16

8y=4x-16

y=1/2 x-2

the slope is 1/2. Since the two lines are perpendicular, the slope of the second line should be -2. Thus, from the point-slope equation we have

y+1=-2(x-5)

y+1=-2x+10

y=-2x+10-1

y=-2x+9

5.

$$x^2 + y^2 + 5x + 6 = 0$$

$$x^2 + 5x + y^2 + 6 = 0$$

We complete the square:

$$x^2 + 2\frac{5}{2}x + \left(\frac{5}{2}\right)^2 - \left(\frac{5}{2}\right)^2 + y^2 + 6 = 0$$

$$\left(x + \frac{5}{2}\right)^2 - \frac{25}{4} + y^2 + 6 = 0$$

$$\left(x + \frac{5}{2}\right)^2 + y^2 - \frac{25}{4} + \frac{24}{4} = 0$$

$$\left(x + \frac{5}{2}\right)^2 + y^2 - \frac{1}{4} = 0$$

$$\left(x + \frac{5}{2}\right)^2 + y^2 = \frac{1}{4}$$

Thus we have a circle with center (-5/2,0) and radius 1/2.

6.

$x=-3y^2+1$ represents a parabola symmetric with the x-axis because the equation is unchanged if we replace y by -y (we have y^2).
-3<0 so the parabola opens to the left.

The vertex is at (1,0).

7.

We have a parabola that opens up (the coefficient of x^2 is =1>0). To find the vertex we complete the square:

$$y = x^2 - 3x + 2$$

$$y = x^2 - 2\frac{3}{2}x + \left(\frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2 + 2$$

$$y = \left(x - \frac{3}{2}\right)^2 - \frac{9}{4} + 2$$

$$y=\left(x-\frac{3}{2}\right)^2-\frac{9}{4}+\frac{8}{4}$$

$$y=\left(x-\frac{3}{2}\right)^2-\frac{1}{4}$$

So the vertex has coordinates x0=3/2 and y0=-1/4.

8.

The equation represents a parabola that opens up. To find the vertex we complete the square:

$$y=3(x^2-5x+4)$$

$$y=3\left(x^2-2\frac{5}{2}x+\left(\frac{5}{2}\right)^2-\left(\frac{5}{2}\right)^2+4\right)$$

$$y=3\left(\left(x-\frac{5}{2}\right)^2-\frac{25}{4}+4\right)$$

$$y=3\left(\left(x-\frac{5}{2}\right)^2-\frac{25}{4}+\frac{16}{4}\right)$$

$$y=3\left(\left(x-\frac{5}{2}\right)^2-\frac{9}{4}\right)$$

$$y=3\left(x-\frac{5}{2}\right)^2-3\frac{9}{4}$$

$$y=3\left(x-\frac{5}{2}\right)^2-\frac{27}{4}$$

Thus the vertex is (5/2, -27/4). To find the y-intercept we make x=0 in $y=3x^{2-}15x+12$ and we get y1=12.

To find the x-intercepts we make y=0 in $y=3x^{2-}15x+12$ and we get $3x^{2-}15x+12 =0$

We solve the quadratic equation using the quadratic formula or factoring.

$$3(x^2-5x+4)=0$$

$$x^2-5x+4=0$$

$$x^2-4x-x+4=0$$

$$x(x-1)-4(x-1)=0$$

$$(x-1)(x-4)=0$$

Thus the x-intercepts are x1=1 and x1=4.

Trigonometry

1. sin (π/6)=......
2. tan(π/4)=........
3. $\sin^2 x+\cos^2 x$=....
4. If cos x=1/3 and 3π/2<x<2π, find sin x.
5. If tan x=1/2 find cot x.
6. If sin x=2/3 and π/2<x<π find sec x.

7. Find the values of x€[0,2π] that satisfy the equation
$|\cot x| = \sqrt{3}$
8. Find the values of x€[0,2π) that satisfy the equation
sin 2x= sin x
9. Find cos x if
$\cos 2x = \frac{7}{9}$

Solutions
1. sin (π/6)=1/2
2. tan(π/4)=1
3. $\sin^2 x+\cos^2 x=1$
4.
Since
$\sin^2 x + \cos^2 x = 1$
we have
$\sin^2 x + \left(\frac{1}{3}\right)^2 = 1$
$\sin^2 x = 1 - \frac{1}{9}$
$\sin^2 x = \frac{9}{9} - \frac{1}{9}$
$\sin^2 x = \frac{8}{9}$
Because 3π/2<x<2π, sinx<0, so

$\sin x = -\frac{\sqrt{8}}{3}$
$\sin x = -\frac{2\sqrt{2}}{3}$
5.
Please memorise the trigonometric formulas!

cot x=1/tan x so cot x=2

6.
Because
$\sin^2 x + \cos^2 x = 1$
we have

$$\left(\frac{2}{3}\right)^2 + \cos^2 x = 1$$

$$\cos^2 x = 1 - \frac{4}{9}$$

$$\cos^2 x = \frac{9}{9} - \frac{4}{9}$$

$$\cos^2 x = \frac{5}{9}$$

Since $\pi/2<x<\pi$, $\cos x<0$ and

$$\cos x = -\frac{\sqrt{5}}{3}$$

$$\sec x = \frac{1}{\cos x} = -\frac{3}{\sqrt{5}}$$

7.

$$|\cot x| = \sqrt{3}$$

$\cot x = \sqrt{3}$ or $\cot x = -\sqrt{3}$

But we know

$$\cot\frac{\pi}{6} = \sqrt{3}$$

so we have

$$x_1 = \frac{\pi}{6}, x_2 = \frac{7\pi}{6}$$

$$x_3 = \frac{5\pi}{6}, x_4 = \frac{11\pi}{6}$$

8.

$$\sin 2x = \sin x$$

$$2\sin x \cos x = \sin x$$

$$\sin x\,(2\cos x - 1) = 0$$

$\sin x = 0$ or $2\cos x - 1 = 0$

$\sin x = 0$ or $\cos x = 1/2$

But

$\sin 0 = 0$ and $\cos\frac{\pi}{3} = 1/2$

So

$$x_1 = 0, x_2 = \pi$$

$$x_3 = \frac{\pi}{3}, x_4 = \frac{5\pi}{3}$$

9.

$\cos 2x = \frac{7}{9}$

$2\cos^2 x - 1 = \frac{7}{9}$

$2\cos^2 x = 1 + \frac{7}{9}$

$2\cos^2 x = \frac{9}{9} + \frac{7}{9}$

$2\cos^2 x = \frac{16}{9}$

$\cos^2 x = \frac{8}{9}$

$cos\ x = -\frac{\sqrt{8}}{3}$ or $cos\ x = \frac{\sqrt{8}}{3}$