Equations and Inequalities

Solve the equations and the inequalities:

1. $2x - 3 = 4x - 1$
2. $x^2 - 3x + 2 = 0$
3. $2x + 3 > 5$
4. $-3x + 1 \leq x + 9$
5. $|x - 3| < 3$
6. $|2x + 3| \geq 1$
7. $|2x + 3| = 7$
8. $x^2 - 4x + 3 < 0$
9. $(x - 3)(2x + 4) \geq 0$
10. $x(x - 2)(x - 5) > 0$

Solutions

1. $2x - 4x = -1 + 3$
   
   $-2x = 2$
   
   $x = 2 / (-2)$
   
   $x = -1$

2. Use the quadratic formula or factoring to solve the equation:

   \[
   \frac{x^2 - 2x - x + 2}{x(x - 1) - 2(x - 1)} = 0
   \]

   $(x - 1)(x - 2) = 0$

   Thus the equation has the solution $x = 1$ and $x = 2$.

3. $2x + 3 > 5$
   
   $2x > 5 - 3$
   
   $2x > 2$
   
   $x > 1$

   $x \in (1, \infty)$

4. $-3x + 1 \leq x + 9$
-3x-x≤9-1
-4x≤8
x≥8/(-4)
x≥-2
x∈[-2,∞)

5. |x-3|<3
-3<x-3<3
-3+3<x<3+3
0<x<6
x∈(0,6)

6. |2x+3|≥1
2x+3≥1 or 2x+3≤-1
2x≥1-3 or 2x≤-1-3
2x≥-2 or 2x≤-4
x≥-1 or x≤-2
x∈(-∞,-2] U [-1,∞)

7. |2x+3|=7
2x+3 =7 or 2x+3=-7
2x=7-3 or 2x=-7-3
2x=4 or 2x=-10
x=2 or x=-5
x∈{-5,2}

8. x²-4x+3<0
We first solve \( x^2 - 4x + 3 = 0 \)
\( x^2 - 3x - x + 3 = 0 \)
\( x(x-1) - 3(x-1) = 0 \)
\( (x-1)(x-3) = 0 \)
\( x = 1 \) or \( x = 3 \)

So the solution of the inequality \( x^2 - 4x + 3 < 0 \)
\( (x-1)(x-3) < 0 \) is
\( x \in (1, 3) \)

9.
We first solve the equation
\( (x-3)(2x+4) = 0 \)
\( x - 3 = 0 \) or \( 2x + 4 = 0 \)
\( x = 3 \) or \( x = -4/2 \)
\( x = 3 \) or \( x = -2 \)

So the solution of \( (x-3)(2x+4) \geq 0 \) is
\( x \in (-\infty, 2] \cup [3, \infty) \)

10.
We first solve the equation \( x(x-2)(x-5) = 0 \); \( x = 0 \) or \( x = 2 \) or \( x = 5 \)

To solve \( x(x-2)(x-5) > 0 \) we make a table

<table>
<thead>
<tr>
<th>( x )</th>
<th>(-\infty)</th>
<th>0</th>
<th>2</th>
<th>5</th>
<th>( +\infty )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x )</td>
<td>( -\infty )</td>
<td>( -\infty )</td>
<td>( -\infty )</td>
<td>( -\infty )</td>
<td>( -\infty )</td>
</tr>
<tr>
<td>( x-2 )</td>
<td>( -\infty )</td>
<td>( 0 )</td>
<td>( 0 )</td>
<td>( 0 )</td>
<td>( 0 )</td>
</tr>
<tr>
<td>( x-5 )</td>
<td>( -\infty )</td>
<td>( -\infty )</td>
<td>( -\infty )</td>
<td>( -\infty )</td>
<td>( -\infty )</td>
</tr>
<tr>
<td>( x(x-2)(x-5) )</td>
<td>( -\infty )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>( -\infty )</td>
</tr>
</tbody>
</table>

and we get \( x(x-2)(x-5) > 0 \) for \( x \in (0, 2) \cup (5, \infty) \)

**Lines, circles, ellipses, hyperbolas, parabolas**

1. Find an equation of the line that passes through \((-1,0)\) and has slope 6.
2. Find an equation of the line with slope 3 and y-intercept 1.
3. Find an equation of the line that passes through \((-1,3)\) and it is parallel to the line \(2x + y = 3\).
4. Find an equation of the line that passes through \((5,-1)\) and it is perpendicular to the line \(4x - 8y = 16\).
5. The equation
\[ x^2 + y^2 + 5x + 6 = 0 \]
represents a ....... with radius R=.... and center with coordinates x_0=...... and y_0=......
6. The equation x=-3y^2+1 represents a ........ symmetric with the ...- axis that opens to the .....  
7. The equation 
y = x^2 - 3x + 2
represents a ....... that opens ...... and has vertex with coordinates x=.... and y=.......  
8. The equation 
y=3x^2+15x+12
represents a ........ with vertex with coordinates x0=..... and y0=...... and x -intercepts x1=..... and x2=....... and y-intercept y1=......

Solutions

1. From the point-slope formula we have
y-0=6(x+1)
y=6x+6

2. From the slope-intercept formula we have
y=3x+1

3. For the line 2x+y=3
y=-2x+3
the slope is -2. Since the two lines are parallel, they have the same slope -2.
From the point-slope equation we have
y-3=-2(x+1)
y=-2x-2+3
y=-2x+1

4. For the line 4x-8y=16
8y=4x-16
y=1/2 x-2
the slope is 1/2. Since the two lines are perpendicular, the slope of the second line should be -2. Thus, from the point-slope equation we have

\[ y+1=-2(x-5) \]
\[ y+1=-2x+10 \]
\[ y=-2x+10-1 \]
\[ y=-2x+9 \]

5.
\[ x^2 + y^2 + 5x + 6 = 0 \]
\[ x^2 + 5x + y^2 + 6 = 0 \]
We complete the square:
\[ x^2 + 2 \cdot \frac{5}{2}x + \left( \frac{5}{2} \right)^2 - \left( \frac{5}{2} \right)^2 + y^2 + 6 = 0 \]
\[ \left( x + \frac{5}{2} \right)^2 - \frac{25}{4} + y^2 + 6 = 0 \]
\[ \left( x + \frac{5}{2} \right)^2 + y^2 - \frac{25}{4} + \frac{24}{4} = 0 \]
\[ \left( x + \frac{5}{2} \right)^2 + y^2 - \frac{1}{4} = 0 \]
\[ \left( x + \frac{5}{2} \right)^2 + y^2 = \frac{1}{4} \]
Thus we have a circle with center (-5/2,0) and radius 1/2.

6.
\[ x=-3y^2+1 \] represents a parabola symmetric with the x-axis because the equation is unchanged if we replace y by -y (we have \( y^2 \)).
\(-3<0\) so the parabola opens to the left.

The vertex is at (1,0).

7.
We have a parabola that opens up (the coefficient of \( x^2 \) is =1>0). To find the vertex we complete the square:
\[ y = x^2 - 3x + 2 \]
\[ y = x^2 - 2 \cdot \frac{3}{2} x + \left( \frac{3}{2} \right)^2 - \left( \frac{3}{2} \right)^2 + 2 \]
\[ y = \left( x - \frac{3}{2} \right)^2 - \frac{9}{4} + 2 \]
\[ y = (x - \frac{3}{2})^2 - \frac{9}{4} + \frac{8}{4} \]
\[ y = (x - \frac{3}{2})^2 - \frac{1}{4} \]
So the vertex has coordinates \( x_0 = \frac{3}{2} \) and \( y_0 = -\frac{1}{4} \).

8.
The equation represents a parabola that opens up. To find the vertex we complete the square:
\[ y = 3(x^2 - 5x + 4) \]
\[ y = 3 \left( x^2 - 2 \cdot \frac{5}{2} x + \left( \frac{5}{2} \right)^2 - \left( \frac{5}{2} \right)^2 + 4 \right) \]
\[ y = 3 \left( x - \frac{5}{2} \right)^2 - \frac{25}{4} + 4 \]
\[ y = 3 \left( x - \frac{5}{2} \right)^2 - \frac{25}{4} + \frac{16}{4} \]
\[ y = 3 \left( x - \frac{5}{2} \right)^2 - \frac{9}{4} \]
\[ y = 3 \left( x - \frac{5}{2} \right)^2 - \frac{9}{4} \]

Thus the vertex is \( (5/2, -27/4) \). To find the y-intercept we make \( x=0 \) in \( y=3x^2-15x+12 \) and we get \( y_1=12 \).
To find the x-intercepts we make \( y=0 \) in \( y=3x^2-15x+12 \) and we get
\[ 3x^2-15x+12 = 0 \]
We solve the quadratic equation using the quadratic formula or factoring.
\[ 3(x^2 - 5x + 4) = 0 \]
\[ x^2 - 5x + 4 = 0 \]
\[ x^2 - 4x - x + 4 = 0 \]
\[ x(x - 1) - 4(x - 1) = 0 \]
\[ (x - 1)(x - 4) = 0 \]
Thus the x-intercepts are \( x_1=1 \) and \( x_1=4 \).

\[ \text{Trigonometry} \]

1. \( \sin \left( \frac{\pi}{6} \right) = ..... \)
2. \( \tan(\pi/4) = ..... \)
3. \( \sin^2 x + \cos^2 x = .... \)
4. If \( \cos x = 1/3 \) and \( 3\pi/2 < x < 2\pi \), find \( \sin x \).
5. If \( \tan x = 1/2 \) find \( \cot x \).
6. If \( \sin x = 2/3 \) and \( \pi/2 < x < \pi \) find \( \sec x \).
7. Find the values of x ∈ [0, 2π] that satisfy the equation
\[ |\cot x| = \sqrt{3} \]
8. Find the values of x ∈ [0, 2π) that satisfy the equation
\[ \sin 2x = \sin x \]
9. Find cos x if
\[ \cos 2x = \frac{7}{9} \]

Solutions
1. \( \sin (\pi/6) = 1/2 \)
2. \( \tan(\pi/4) = 1 \)
3. \( \sin^2 x + \cos^2 x = 1 \)
4. Since
\[ \sin^2 x + \cos^2 x = 1 \]
we have
\[ \sin^2 x + \left(\frac{1}{3}\right)^2 = 1 \]
\[ \sin^2 x = 1 - \frac{1}{9} \]
\[ \sin^2 x = \frac{9}{9} - \frac{1}{9} \]
\[ \sin^2 x = \frac{8}{9} \]
Because \( 3\pi/2 < x < 2\pi \), \( \sin x < 0 \), so
\[ \sin x = -\frac{\sqrt{8}}{3} \]
\[ \sin x = -\frac{2\sqrt{2}}{3} \]
5. Please memorise the trigonometric formulas!
\[ \cot x = 1/\tan x \] so \( \cot x = 2 \)
6. Because
\[ \sin^2 x + \cos^2 x = 1 \]
we have
\[
\left(\frac{2}{3}\right)^2 + \cos^2 x = 1
\]

\[\cos^2 x = 1 - \frac{4}{9}\]

\[\cos^2 x = \frac{9}{9} - \frac{4}{9}\]

\[\cos^2 x = \frac{5}{9}\]

Since \(\pi/2 < x < \pi\), \(\cos x < 0\) and

\[\cos x = -\frac{\sqrt{5}}{3}\]

\[\sec x = \frac{1}{\cos x} = -\frac{3}{\sqrt{5}}\]

7.

| \cot x | = \sqrt{3}

\[\cot x = \sqrt{3} \text{ or } \cot x = -\sqrt{3}\]

But we know

\[\cot \frac{\pi}{6} = \sqrt{3}\]

so we have

\[x_1 = \frac{\pi}{6}, x_2 = \frac{7\pi}{6}\]

\[x_3 = \frac{5\pi}{6}, x_4 = \frac{11\pi}{6}\]

8.

\[\sin 2x = \sin x\]

\[2 \sin x \cos x = \sin x\]

\[\sin x (2 \cos x - 1) = 0\]

\[\sin x = 0 \text{ or } 2 \cos x - 1 = 0\]

\[\sin x = 0 \text{ or } \cos x = 1/2\]

But

\[\sin 0 = 0 \text{ and } \cos \frac{\pi}{3} = 1/2\]

So

\[x_1 = 0, x_2 = \pi\]

\[x_3 = \frac{\pi}{3}, x_4 = \frac{5\pi}{3}\]

9.
\[
\begin{align*}
\cos 2x &= \frac{7}{9} \\
2 \cos^2 x - 1 &= \frac{7}{9} \\
2 \cos^2 x &= 1 + \frac{7}{9} \\
2 \cos^2 x &= \frac{9}{9} + \frac{7}{9} \\
2 \cos^2 x &= \frac{16}{9} \\
\cos^2 x &= \frac{8}{9} \\
\cos x &= \frac{-\sqrt{8}}{3} \text{ or } \cos x = \frac{\sqrt{8}}{3}
\end{align*}
\]